1/8 CORRECTION FACTOR OF SCHAWLOW-TOWNES LIMIT IN FM NOISE OF NEGATIVE FREQUENCY FEEDBACK LASERS

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Abstract

A limit of the frequency modulation (FM) noise in a negative frequency feedback laser using a Fabry-Perot interferometer (FP) as a frequency discriminator is equal to the modified Schawlow-Townes limit of the laser having the same Fabry-Perot resonator by a correction factor. We show that this correction factor is 1/8 which results from no gain noise, half vacuum noise and doubled energy of resonator as compared with a laser resonator.

1. Introduction

A limit of the frequency modulation (FM) noise in a negative frequency feedback laser using a Fabry-Perot interferometer (FP) as an optical frequency discriminator has been well known. The limit, which is determined by the zero-point field fluctuations introduced by the discriminator and the power divider[1], is equal to the modified Schawlow-Townes limit of the laser having the same Fabry-Perot resonator by a correction factor which is ambiguous for various ways of parameter definitions. This correction factor is important for practical use of such a laser in a gravitational wave detection, coherent optical transmission system, and so on. We point out that this factor is 1/8 which results from no gain noise, half vacuum noise and doubled energy of resonator as compared with a laser resonator.

2. Schawlow-Townes limit of FM noise

When we we can ignore the relaxation oscillation in a laser, we have

$$da/dt = -(i\omega_c + (r_c - G(n))/2)a + \sqrt{r_c}(f_i + f_g), \quad f_o = \sqrt{r_c}a - f_i, \tag{1}$$

$$[a, a^+] = 1, \quad [f_i, f_i^+] = [f_o, f_o^+] = \delta(t-s), \quad r_c[f_g, f_g^+] = -(dRe(nG)/dn)\delta(t-s)$$

where ω_c, r_c, a and n are an angular frequency, a power damping rate, an annihilation operator and a photon number a^+a of the resonator's quasi-mode respectively. G is a gain, f_g is the gain noise, f_i is an incoming photon flux and f_o is an outgoing photon flux[2]. Furthormore, if f_i is in a vacuum state, the so-called α parameter is zero and a pumping level is high, then we get

$$<Re^2(f_i)> = <Im^2(f_i)> = <Re^2(f_g)> = <Im^2(f_g)> = 1/4,$$

$$<dRe(nG)/dn> = 0, \quad <dIm(G)/dn> = 0$$

where Re (Im) is a real (imaginary) part, and the averaged value $<XY>$ means

$$<X(t)Y(s)> = <XY>\ \delta(t-s).$$

Fig.1(a) shows a laser using a FP resonator with two output ports, and f_i and f_o in eq.(1) are

$$f_i = (f_{v1} + f_{v2})/\sqrt{2}, \quad f_o = (f_{o1} + f_{o2})/\sqrt{2}.$$

For small fluctuations of $(A, F_i, F_o, F_g) := exp(i\omega t)(a, f_i, f_o, f_g)$ where ω is an optical angular frequency(':=' means a definition), we can linearize eq.(1) about the steady-state mean values, and get

$$A = \sqrt{n}e^{-i\phi} \approx <A> (1 + \delta n / <2n> - i\delta\phi), \quad F_o = \sqrt{N}e^{-i\Psi} \approx <F_o> (1 + \delta N / <2N> - i\delta\Psi), \quad (2)$$

$$d\delta\phi/dt = -(r_c/n)^{1/2} Im(F_i + F_g), \quad \delta\Psi = \delta\phi + (r_c n)^{-1/2} Im(F_i) \quad (3)$$

where we take $<A>$ as real(δ means a fluctuation part). Therefore, the side-band power spectrum S_ω of the angular frequency fluctuation $d\delta\Psi/dt$ of out-going light is

$$S_\omega = <N>^{-1} (r_c^2 + (2\pi\nu)^2/2)d\nu \approx (\hbar\omega/P_o)r_c^2 d\nu \quad (2\pi\nu << r_c) \quad (4)$$

where $\hbar\omega$ is an energy of a photon, P_o is a power of the out-going light of the laser, and ν is a Fourier frequency. The $r_c^2/ < N >$ is called the Schawlow-Towns limit of FM noise revised by [3], and the $(2\pi\nu)^2/ < 2N >$ is the so-called shot noise which can be ignored when $2\pi\nu << r_c$. The intensity fluctuation δN, which is a shot noise, can be also ignored when $2\pi\nu << r_c$ because the power spectrum of $d(\delta N/ <2N>)/dt$ in eq.(2) is $(2\pi\nu)^2/ <2N>$.

3. Limit of FM noise in a negative frequency feedback laser

Fig.1(b) shows a negative frequency feedback laser with detection of the reflected light from a FP resonator which is used as an optical frequency discriminator. The laser's angular frequency ω is locked to ω_0 using a slope of resonant profile of the FP where ω_0 is slightly detuned from the FP's resonant angular frequency ω_c. The output current $i(t)$ of the photo detector with the unit quantum efficiency is

$$i(t) = (eP_i/\hbar\omega)/(1 + r_c^2/4(\omega - \omega_c)^2) + i_n(t) \approx (eP_i/\hbar\omega)(2\Delta\omega_c/r_c)^2(1 - 2\Delta\omega/\Delta\omega_c) + i_n(t) \quad (5)$$

if $r_c >> |\Delta\omega_c| >> |\Delta\omega|$ where $\Delta\omega_c := \omega_c - \omega_0, \Delta\omega := \omega - \omega_0$, r_c is the FP's power damping rate, P_i is a power of in-coming light to FP, $i_n(t)$ is a shot noise by photo-electrons, and e is a electron charge. We lock ω to ω_0 by maintainig $i(t)$ constant, and have

$$\Delta\omega \approx (\Delta\omega_c/ <2i>)i_n, \quad S_\omega \approx (\hbar\omega/8P_i)r_c^2 d\nu = \text{(Schawlow-Townes's limit)}/8 \quad (6)$$

because the side-band power spectrum of shot noise i_n is $2e<i>d\nu$.

In order to explain why the factor 1/8 appears in eq.(6), we re-calculate the power spectrum S_ω of FM noise by using quantum theory. In Fig.1(b), f_v is a photon flux(zero-point field fluctuation) incident on one mirror of the FP resonator. By the same definitions as above, we have

$$dA/dt = -(i\Delta\omega_c + r_c/2)A + \sqrt{r_c}(F_i + F_v)/\sqrt{2} \quad (A := exp(i\omega_0 t)a, \quad etc), \quad (7)$$

$$F_r = \sqrt{r_c/2}A - F_i \approx 2i(\Delta\omega - \Delta\omega_c) <F_i> /r_c + F_v \quad (8)$$

$$(\Delta\omega := d\delta\Psi/dt, \quad F_i \approx <F_i> (1 - i\delta\Psi), \quad P_i = \hbar\omega|<F_i>|^2),$$

$$i(t) = eF_r^+ F_r \approx (eP_i/\hbar\omega)(2\Delta\omega_c/r_c)^2(1 - (2\Delta\omega + <r_c/F_i> Im(F_v))/\Delta\omega_c) \quad (9)$$

where we take $<F_i>$ as real(we ignored the intensity fluctuation δN). Eq.(8) is derived from

$$F_r = -\sqrt{2/r_c}(d/dt + i\Delta\omega_c)A + F_v, \quad A = \sqrt{2/r_c}(F_i + F_v) - (2/r_c)(d/dt + i\Delta\omega_c)A$$

in which F_v and $(d/dt + i\Delta\omega_c)$ are small quantities. By maintaining $i(t)$ constant, we have

$$\Delta\omega \approx - <r_c/2F_i> Im(F_v), \quad S_\omega = (\hbar\omega/8P_i)r_c 2d\nu = \text{(Schawlow-Townes's limit)}/8 \quad (10)$$

which is equal to eq.(6) in a classical treatment. Deleting the F_i in eq.(7), we get a following equation

$$d\delta\phi/dt = -(r_c/2n)^{1/2} Im(F_v), \quad r_c|<A>|^2 = 2|<F_i>|^2 \quad (11)$$

which corresponds to eq.(3) in case of a laser, and is derived from

$$-r_cA/2+\sqrt{r_c/2}F_i = -\sqrt{r_c/2}F_r \approx 2i <F_i> \Delta\omega_c/r_c - Re(F_v).$$

Eq.(11) explains that the correction factor 1/8 in eq.(10) results from no gain noise, half vacuum noise and doubled energy of resonator as compared with a laser resonator.

Even if Fabry-Perot resonator has one output port, we can realize the same factor 1/8 by combining Mach-Zehnder interferometer and balanced detector with the FP resonator as Fig.1(c). Although we must adjust the arm length difference of the interferometer, we can lock the laser frequency to the center of a resonant spectral profile of the FP. From Fig.1(c), we have

$$dA/dt = -r_cA/2+\sqrt{r_c}F_1, \quad F_r = \sqrt{r_c}A - F_1 \quad (F_1 := (F_i+F_v)/\sqrt{2}, \quad F_2 := i(F_i-F_v)/\sqrt{2}),$$
$$i(t) = e(F_3^+F_3 - F_4^+F_4), \quad (F_3 := (F_r+F_2)/\sqrt{2}, \quad F_4 := (F_r-F_2)/\sqrt{2}). \tag{12}$$

With the linearization and the Fourier series analysis[2], Eq.(12) leads to

$$F_i = T^{-1/2}\sum a_k exp(-i\Delta\omega_k t) = <F_i> (1+\delta N/ <2N> -i\delta\Psi), \quad [a_k, a_k^+] = 1,$$
$$F_v = T^{-1/2}\sum v_k exp(-i\Delta\omega_k t), \quad [v_k, v_k^+] = 1, \quad <v_k^+ v_k> = 0 \quad (\Delta\omega_k := \omega_k - \omega_c),$$
$$i(t) = T^{-1/2}\sum <eF_i/2i> (r_c/2 - i\Delta\omega_k)^{-1}(2i\Delta\omega_k(a_k - a_{k'}^+) + r_c(v_k - v_{k'}^+))exp(-i\Delta\omega_k t) \tag{13}$$

where we take $<F_i>$ as real, and $\Delta\omega_{k'} := -\Delta\omega_k$. We control $\delta\Psi$ to maintain $i(t)$ constant, and have

$$\delta\Psi_k = (a_k - a_{k'}^+)/ < 2iF_i > \approx r_c(v_k - v_{k'}^+)/ <4\Delta\omega_k F_i> \quad (\delta\Psi = T^{-1/2}\sum \delta\Psi_k exp(-i\Delta\omega_k t)),$$
$$S_\omega = 2r_c^2 <Im^2 F_v> d\nu/ <4N> = r_c^2 d\nu/ <8N> . \tag{14}$$

In eq.(14), the correction factor of the same 1/8 appears again and $S_\omega \approx 0$ if F_v is in a squeezed vacuum-state ($<Im^2F_v> \approx 0$). It is also remarkable that the FM noise of F_i has no shot noise component. Therefore the fluctuation of $Im(F_i)$ is below the shot noise limit at $2\pi\nu >> r_c$(squeezing. see appendix), while we can not take it out from this feedback system.

Finally, we point out that FM modulation method with FP resonator having one output port also leads to the same correction factor 1/8 as above when the modulation index is nearly zero. A method using a $\lambda/4$ plate in the FP resonator which does not need any modulation techniques and adjustments of optical path-length is known[4], and leads to the same factor 1/8.

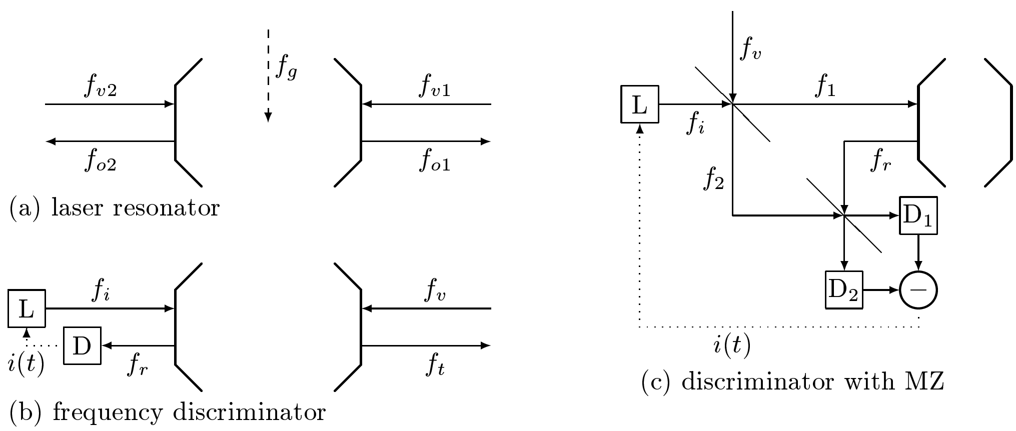

図 1: Photon flux at FP resonator

Appendix:FM noise and squeezing

The power spectrum of phase fluctuation is also the squeezing spectrum. In fact, for a photon flux $F(t)$, we have with the linearization and the Fourier series analysis

$$F(t) := T^{-1/2} \sum a_k exp(-i\Delta\omega_k t) \approx <F> (1 + \delta N/ <2N> -i\delta\Psi), \quad [a_k, a_k^+] = 1.$$

$$\delta N_k = <F>^* a_k + <F> a_{k'}^+, \quad \delta\Psi_k = (<F>^* a_k - <F> a_{k'}^+)/ <2iN> \quad (\Delta\omega_{k'} := -\Delta\omega_k).$$

$$X(t) := Re(e^{-i\theta}\Delta F) = T^{-1/2} \sum x_k exp(-i\Delta\omega_k t), \quad x_k := (e^{-i\theta} a_k + e^{i\theta} a_{k'}^+)/2 = x_{k'}^+ \quad (k \neq 0).$$

A squeezing spectrum $S(\Delta\omega_k, e^{i\theta})$ which is defined as a Fourier transform of $<: X(t)X(0) :>$ [5], and a side-band power spectrum S_ω of FM noise are

$$S(\Delta\omega_k, e^{i\theta}) = <: x_k x_{k'} :> = <x_k x_{k'}> -1/4,$$

$$S_\omega = 2 <|\Delta\omega_k \delta\Psi_k|^2> d\nu = 2 <\Delta\omega_k^2/N> (1/4 + S(\Delta\omega_k,\ i<F>/|<F>|)) d\nu.$$

Squeezed state can be generated by a degenerate parametric oscillator(DPO). However if we pump it by a laser light which has a large FM noise, then the FM noise of output light is not reduced. In fact, as is shown by ref.[6], we have Langevin equations and its linearizations such as

$$dA_1/dt = -r_1 A_1/2 + gA_1^+ A_2 + \sqrt{r_1} F_{i1}, \quad dA_2/dt = -r_2 A_2/2 - g^* A_1^2/2 + \sqrt{r_2} F_{i2},$$

$$F_{o1} = \sqrt{r_1} A_1 - F_{i1}, \quad F_{o2} = \sqrt{r_2} A_2 - F_{i2}.$$

$$<N_{i2}> = (r_1 + \mu n_1)^2/8\mu, \quad <N_{o1}> = r_1 n_1 \quad (\mu := 2|g|^2/r_2, \quad n_1 := <A_1^+ A_1>),$$

$$Im(\delta F_{o1}) = Im(-\mu n_1 F_{i1} + \sqrt{2 r_1 \mu n_1} \delta F_{i2})/(\mu n_1 + r_1) \quad (|\Delta\omega_k| << r_1, r_2)$$

where F_{i2} is a pumping flux($\omega_2 = 2\omega_1$). Therefore, when F_{i2} has a large FM noise, we get

$$<\delta\Psi_{o1}^2> = K/4\Delta\omega_k^2 + (1 - r_1^2/(\mu n_1 + r_1)^2)/ <4N_{o1}> \quad (<\delta\Psi_{i2}^2> := K/\Delta\omega_k^2 + 1/ <4N_{i2}>)$$

which means that we should use a DPO as a vacuum squeezer ($<Im^2 F_{o1}> = <N_{o1}\delta\Psi_{o1}^2> \approx 0$) when we pump a DPO by a light with large FM noise. The squeezed vacuum is valuable because the classical shot noise usually results from one quadrature component of zero-point fileld fluctuation of the vacuum.

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